Sequences & Series

Short Type Questions

1. The first term of an A.P. is a, and the sum of the first p terms is zero, show that the sum of its next q terms is $\dfrac{-a(p+q)q}{p-1}$. [**Hint:** Required sum $= S_{p+q} - S_p$]

2. A man saved Rs 66000 in 20 years. In each succeeding year after the first year he saved Rs 200 more than what he saved in the previous year. How much did he save in the first year?

3. A man accepts a position with an initial salary of Rs 5200 per month. It is understood that he will receive an automatic increase of Rs 320 in the very next month and each month thereafter.

   (a) Find his salary for the tenth month

   (b) What is his total earnings during the first year?

4. If the pth and qth terms of a G.P. are q and p respectively, show that its $(p+q)^{th}$ term is $\left(\dfrac{q^p}{p^q}\right)^{\frac{1}{p-q}}$.

5. A carpenter was hired to build 192 window frames. The first day he made five frames and each day, thereafter he made two more frames than he made the day before. How many days did it take him to finish the job?

6. We know the sum of the interior angles of a triangle is $180°$. Show that the sums of the interior angles of polygons with 3, 4, 5, 6, ... sides form an arithmetic progression. Find the sum of the interior angles for a 21 sided polygon.

7. A side of an equilateral triangle is 20cm long. A second equilateral triangle is inscribed in it by joining the mid points of the sides of the first triangle. The process is continued as shown in the accompanying diagram. Find the perimeter of the sixth inscribed equilateral triangle.

8. In a potato race 20 potatoes are placed in a line at intervals of 4 metres with the first potato 24 metres from the starting point. A contestant is required to bring the potatoes back to the starting place one at a time. How far would he run in bringing back all the potatoes?

9. In a cricket tournament 16 school teams participated. A sum of Rs 8000 is to be awarded among themselves as prize money. If the last placed team is awarded Rs 275 in prize money and the award increases by the same amount for

successive finishing places, how much amount will the first place team receive?

10. If $a_1, a_2, a_3, ..., a_n$ are in A.P., where $a_i > 0$ for all i, show that

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + ... + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}} = \frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$$

11. Find the sum of the series
    $(3^3 - 2^3) + (5^3 - 4^3) + (7^3 - 6^3) + ...$ to (i) n terms (ii) 10 terms

12. Find the rth term of an A.P. sum of whose first n terms is $2n + 3n^2$.
    [Hint: $a_n = S_n - S_{n-1}$]

13. If A is the arithmetic mean and G_1, G_2 be two geometric means between any two numbers, then prove that

$$2A = \frac{G_1^2}{G_2} + \frac{G_2^2}{G_1}$$

14. If $\theta_1, \theta_2, \theta_3, ..., \theta_n$ are in A.P., whose common difference is d, show that

$$\sec\theta_1 \sec\theta_2 + \sec\theta_2 \sec\theta_3 + ... + \sec\theta_{n-1} \sec\theta_n = \frac{\tan\theta_n - \tan\theta_1}{\sin d}.$$

15. If the sum of p terms of an A.P. is q and the sum of q terms is p, show that the sum of $p + q$ terms is $-(p + q)$. Also, find the sum of first $p - q$ terms $(p > q)$.

16. If p^{th}, q^{th}, and r^{th} terms of an A.P. and G.P. are both a, b and c respectively, show that

$$a^{b-c} \cdot b^{c-a} \cdot c^{a-b} = 1$$

Objective Type Questions

17. If the sum of n terms of an A.P. is given by
    $S_n = 3n + 2n^2$, then the common difference of the A.P. is

(A) 3   (B) 2   (C) 6   (D) 4

18. The third term of G.P. is 4. The product of its first 5 terms is

(A) 4^3 (B) 4^4 (C) 4^5 (D) None of these

19. If 9 times the 9th term of an A.P. is equal to 13 times the 13th term, then the 22nd term of the A.P. is

(A) 0   (B) 22   (C) 220   (D) 198

20. If $x, 2y, 3z$ are in A.P., where the distinct numbers x, y, z are in G.P. then the common ratio of the G.P. is

(A) 3   (B) $\dfrac{1}{3}$ (C) 2   (D) $\dfrac{1}{2}$

21. If in an A.P., $S_n = q\, n^2$ and $S_m = qm^2$, where S_r denotes the sum of r terms of the A.P., then S_q equals

(A) $\dfrac{q^3}{2}$ (B) mnq (C) q^3 (D) $(m+n)\, q^2$

22. Let S_n denote the sum of the first n terms of an A.P. If $S_{2n} = 3S_n$ then $S_{3n} : S_n$ is equal to

(A) 4   (B) 6   (C) 8   (D) 10

23. The minimum value of $4^x + 4^{1-x}$, $x \in R$, is

(A) 2   (B) 4   (C) 1   (D) 0

24. Let S_n denote the sum of the cubes of the first n natural numbers and s_n denote the sum of the first n natural numbers. Then $\displaystyle\sum_{r=1}^{n} \dfrac{S_r}{s_r}$ equals

(A) $\dfrac{n(n+1)(n+2)}{6}$ (B) $\dfrac{n(n+1)}{2}$

(C) $\dfrac{n^2 + 3n + 2}{2}$ (D) None of these

25. If t_n denotes the nth term of the series $2 + 3 + 6 + 11 + 18 + \dots$ then t_{50} is

(A) $49^2 - 1$ (B) 49^2 (C) $50^2 + 1$ (D) $49^2 + 2$

26. The lengths of three unequal edges of a rectangular solid block are in G.P. The volume of the block is 216 cm^3 and the total surface area is 252cm^2. The length of the longest edge is

(A) 12 cm   (B) 6 cm   (C) 18 cm   (D) 3 cm